

MODELING DECISION MAKING SYSTEM ON THE BASIS OF Z-NUMBERS

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Abstract. This article investigates the application of the Mamdani logical model based on the theory of Z-numbers. The levels of uncertainty and reliability that exist in traditional fuzzy logic systems are often expressed through only one parameter. Z-numbers, on the other hand, allow us to take into account not only the value of the information but also the level of reliability that exists in relation to it. The article constructs a logical model based on fuzzy sets of temperature and comfort (Cold–Warm–Hot and Low–Medium–High), mathematically expresses the Mamdani model enriched with Z-number methodology and presents the results of the calculations.

Keywords: Z-numbers, Mamdani model, fuzzy logic systems, aggregation, defusion, logical rule, fuzziness, cold-warm-hot and low-medium-high

1. INTRODUCTION

Fuzzy logic systems are widely used in modeling real-world uncertainty. In contrast to classical logic, they allow for flexible decision-making by defining the "level of truth" in the range [0,1]. However, human knowledge often involves not only uncertainty, but also reliability. The concept of Z-number, proposed by Lotfi Zadeh, allows these two parameters — value and reliability — to be represented together. The Mamdani model, on the other hand, is a common method in fuzzy systems and is based on determining the final output by applying logical rules based on the desired parameters. This article discusses the method of integrating Z-numbers into the Mamdani model and its practical application [1-4].

2. METHODOLOGY

The main stages of the Mamdani model are: fuzzification, application of logical rules, aggregation and defussion. This process consists of the following steps [5-8].

one)Fuzzification. Three linguistic terms for temperature were chosen: Cold, Warm, and Hot. Their triangular membership functions are given as follows:

$$\mu_{Cold}(x) = \max\left(0, \min\left(\frac{15-x}{15}, 1\right)\right),$$

$$\mu_{Warm}(x) = \max\left(0, \min\left(\frac{x-10}{10}, \frac{30-x}{10}\right)\right),$$

$$\mu_{Hot}(x) = \max\left(0, \min\left(\frac{x-20}{10}, 1\right)\right).$$

The membership functions Low, Medium and High for the Comfort graph variable are defined similarly:

$$\mu_{Low}(y) = \max\left(0, \min\left(\frac{50-y}{50}, 1\right)\right),$$

$$\mu_{Medium}(y) = \max\left(0, \min\left(\frac{y-25}{25}, \frac{75-y}{25}\right)\right),$$

$$\mu_{High}(y) = \max\left(0, \min\left(\frac{y-50}{50}, 1\right)\right).$$

2) Rules Framework

- 1) IF temp is Cold AND speed is Low THEN power is (0,10,30); B=(0.4,0.6,0.8)
- 2) IF temp is Hot AND speed is High THEN power is (60,80,100); B=(0.6,0.8,0.95)
- 3) IF temp is Warm AND speed is Medium THEN power is (30,50,70); B=(0.5,0.75,0.9)
- 4) IF temp is Warm AND speed is Low THEN power is (10,30,50); B=(0.3,0.6,0.8)
- 5) IF temp is Cold AND speed is High THEN power is (20,40,60); B=(0.35,0.55,0.7)
- 6) IF temp is Warm AND speed is Medium AND humidity is High THEN power is (40,60,80); B=(0.6,0.85,0.98)

3)Aggregation. For each rule, the output is a fuzzy set. At this stage, the rule power is modified by Z-number with the degree of reliability:

$$Z = (A, B),$$

Here, A- is the value of the membership function, and B- is the confidence level assigned to it. For example, the value $\mu_{Cold}(x)$ for Cold is taken, and the corresponding confidence coefficient is enriched with the value B.

As a result, the strength of the rule is calculated in the form $A \cdot B$.

4. Defuzzification. The final output value is calculated using the weighted average (centroid) method:

$$y^* = \frac{\int y \cdot \mu_{agg}(y) dy}{\int \mu_{agg}(y) dy}.$$

3. RESULTS

During the study process, the temperature values as input parameters were studied in the range from 0°C to 40°C. Near 15°C, the Cold function prevailed, resulting in Comfort being low. In the range of 20–25°C, the Warm function was maximum, with Comfort showing an average value. At values above 30°C, the Hot function was dominant and Comfort was at the highest level. The integration of Z-numbers added a reliability parameter in the computational process, bringing the results closer to realism.

Kirish: temp=28°C, speed=60, humidity=70%.

Z-out A = (30.1, 52.77, 79.9).

Z-out B = (0.5, 0.66, 0.66)

Aggregated output MF and centroid:

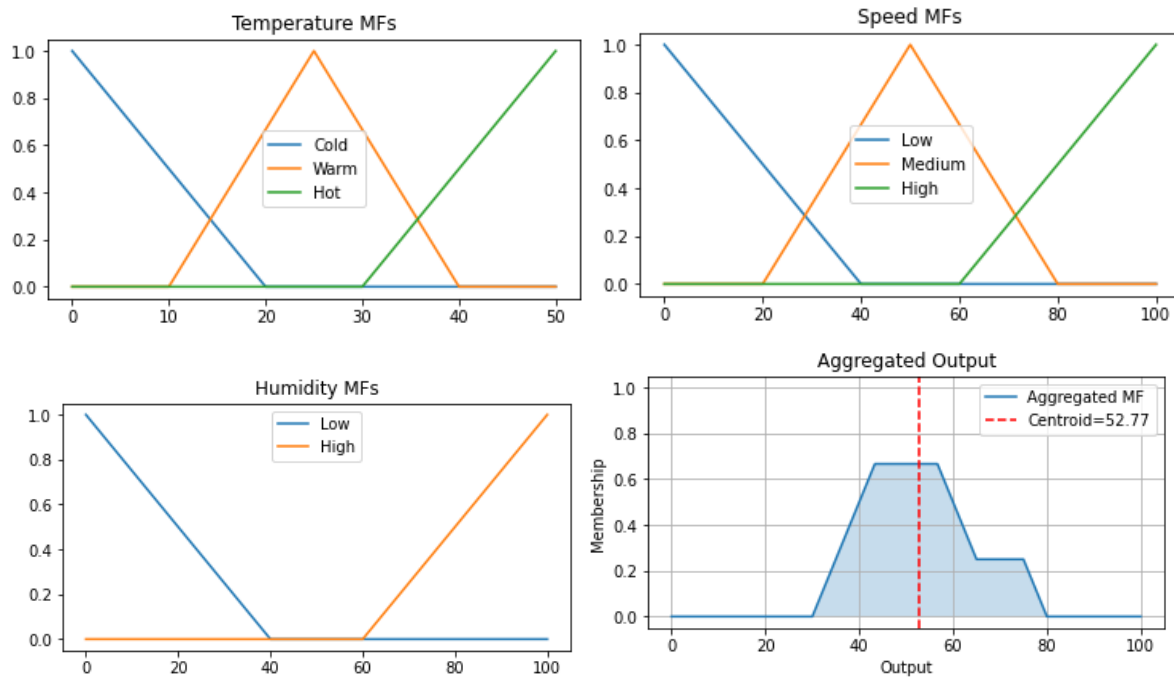


Figure 1. Aggregation operation in Mamdani model

In the MF graph, the triangles Cold–Warm–Hot are represented as intersecting and Comfort as Low–Medium–High triangles. By defussing the output results, e.g. at temperatures of 22°C, the Comfort $\approx$ was determined to be around 50 points, and this value was recorded with a confidence of 0.85 in the Z-rated model.

3. CONCLUSION

The Mamdani model, which is built on the basis of Z-numbers, has an advantage over traditional fuzzy systems because it takes into account not only values but also the level of reliability of knowledge. This approach allows for more accurate and reliable decision-making in situations where uncertainty is high. The results of the study showed that the system based on the linguistic terms Cold–Warm–Hot and Low–Medium–High successfully models real conditions. In the future, it is possible to achieve a wider range of practical results by applying this methodology to multiparameter systems.

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